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GRAVITY-INDUCED MOTION OF A UNIFORMLY HEATED SOLID PARTICLE IN A GASEOUS MEDIUM

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The steady motion of a uniformly heated spherical aerosol particle in a viscous gaseous medium is analyzed in the Stokes approximation under the condition that the mean temperature of the particle surface can be substantially different from the ambient temperature. An analytical expression for the drag force and the velocity of gravity-induced motion of the uniformly heated spherical solid particle is derived with allowance for temperature dependences of the gaseous medium density, viscosity, and thermal conductivity. It is numerically demonstrated that heating of the particle surface has a significant effect on the drag and velocity of gravity-induced motion.

Key words: *Stokes problem, gas–particle system, nonisothermality.*

Introduction. Knowing the force of medium resistance to particle motion is necessary in solving many scientific and engineering problems, e.g., in designing experimental facilities where directed motion of particles has to be ensured, in developing methods of fine cleaning of gases from aerosol particles, in analyzing particle precipitation in plane-parallel channels with different temperatures, and in solving numerous problems related to cleaning of industrial wastes from aerosol particles. It is known that the drag force can be substantially changed by deliberate heating of the particle, for instance, in the laser radiation field in the case of clearing of clouds and fogs, in combustion of particles in chemically active media, and in particle transfer in photoprecipitators where the particle motion occurs under conditions of substantial relative differences in temperature. The relative difference in temperature is understood as the difference between the particle surface temperature and the temperature of the medium far from the particle. The relative difference in temperature is assumed to be small if the inequality $(T_{eS} - T_{e\infty})/T_{e\infty} \ll 1$ holds and to be large if $(T_{eS} - T_{e\infty})/T_{e\infty} \sim 0(1)$ (T_{eS} is the mean temperature of the particle surface and $T_{e\infty}$ is the temperature of the gaseous medium far from the particle). The heated surface of the aerosol particle exerts a significant effect on thermophysical characteristics of the ambient gaseous medium and, as a consequence, on the magnitude of the drag force.

The problem of the drag force acting on a heated spherical solid particle was first solved in [1], though the results obtained there cannot be used for large differences in temperature because of the inappropriate choice of the method used to solve gas-dynamic equations. Moreover, Kassoy et al. [1] considered only the linear dependence of thermal conductivity and dynamic viscosity on temperature.

For viscosity, thermal conductivity, and diffusion described by power functions [2], an analytical solution of the problem of the drag force acting on a heated spherical particle was derived in [3–6]; results obtained in these papers allow the estimates to be made at large differences in temperature.

Sometimes solving problems with nonisothermal flows additionally involved the allowance for the so-called Barnett temperature stresses [7]; for example, the problem of the flow around a strongly heated sphere was solved numerically in [8] and analytically in [9]. The Barnett stresses can produce a significant effect on the motion of particles at Mach numbers $\mathrm{M} \to 0$ [8]. In the present paper, the motion of particles is considered for rather low

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Knudsen numbers and not too low Mach numbers, where the temperature stresses may be neglected even for relative differences in temperature of the order of unity. It should be noted that the solutions of differential equations that describe the velocity and pressure fields in the form of power series, which are obtained by the method reducing the problem dimension, are rather cumbersome. In the present work, the solution of gas-dynamic equations is found in the form of generalized power series, which substantially simplifies the calculations.

1. Formulation of the Problem. We consider a gaseous flow around a uniformly heated spherical aerosol particle with substantial differences in temperature in the vicinity of the particle.

The particle can be heated to high temperatures, for instance, in the field of laser radiation. The temperature distribution over the particle surface is close to uniform if the thermal conductivity of the particle is much higher than the thermal conductivity of the gas or if the radiation wavelength is substantially greater than the particle radius [10].

In analyzing the flow around the aerosol particle, we assume that all processes are quasi-stationary because of the small time of thermal relaxation of the gas–particle system. The particle motion occurs at Peclet and Reynolds numbers much smaller than unity. As $(T_{eS} - T_{e\infty})/T_{e\infty} \sim 0(1)$, we have to take into account the temperature dependence of viscosity and thermal conductivity of the gaseous medium. In the present work, we use the power dependences [2]

$$\mu_e = \mu_{e\infty}(T_e/T_{e\infty})^{\beta}, \qquad \lambda_e = \lambda_{e\infty}(T_e/T_{e\infty})^{\alpha}, \qquad 0.5 \leqslant \alpha \leqslant 1.0, \qquad 0.5 \leqslant \beta \leqslant 1.0,$$

where $\mu_{e\infty} = \mu_e(T_{e\infty})$, $\lambda_{e\infty} = \lambda_e(T_{e\infty})$, μ_e and λ_e are the dynamic viscosity and thermal conductivity of the gaseous medium, respectively, and T_e is the gas temperature. The particle is assumed to be uniform in terms of its composition and large (Knudsen number $\mathrm{Kn} = \lambda/R \ll 0.01$, where λ is the mean free path of gas molecules), and there are no phase transitions on the particle surface. The particle radius is rather small, and the influence of gravity-induced convection on the temperature distribution may be neglected.

Under the assumptions made, the equations and boundary conditions for velocity U_e, pressure P_e, and temperature T_e in the Stokes approximations have the following form [11, 12]:

$$\frac{\partial P_e}{\partial x_k} = \frac{\partial}{\partial x_j}\Big[\mu_e\Big(\frac{\partial U_k^e}{\partial x_j} + \frac{\partial U_j^e}{\partial x_k} - \frac{2}{3}\,\delta_{jk}\,\frac{\partial U_m^e}{\partial x_m}\Big)\Big] + \boldsymbol{F}_k^{(mg)}, \qquad \operatorname{div}(\rho_e U_e) = 0; \tag{1.1}$$

$$\operatorname{div}(\lambda_e \nabla T_e) = 0;$$

$$r = R, \qquad U_r^e = 0, \qquad U_\theta^e = 0, \qquad T_e = T_{eS}; \tag{1.2}$$

$$r \to \infty, \qquad \boldsymbol{U}_e \to U_\infty \boldsymbol{e}_r \cos\theta - U_\infty \boldsymbol{e}_\theta \sin\theta, \qquad T_e \to T_{e\infty}, \qquad P_e \to P_{e\infty}. \tag{1.3}$$

Here U_r^e and U_θ^e are the radial and tangential components of the mass velocity of the gas in a spherical coordinate system $\boldsymbol{U}_e$, ρ_e is the density of the gaseous medium, $U_\infty = |\boldsymbol{U}_\infty|$ ($\boldsymbol{U}_\infty$ is the free-stream velocity determined from the condition of vanishing of the total force acting on the particle), $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are the unit vectors of the spherical coordinate system, and $\boldsymbol{F}^{(mg)}$ is the vector of the gravity force ($\boldsymbol{U}_\infty$ and $|\boldsymbol{F}^{(mg)}|$ are related so that the total force acting on the particle equals zero).

The no-slip conditions for the normal and tangential components of mass velocity and the condition of a constant temperature are set on the particle surface. Conditions (1.3) are used as the boundary conditions at infinity, i.e., far from the particle. Hereinafter, the index e refers to the gas, and the index ∞ refers to physical quantities characterizing the ambient medium in the undisturbed flow.

2. Velocity and Temperature Fields. Velocity of Gravity-Induced Falling of the Particle. To determine the force acting on a uniformly heated aerosol particle and the velocity of its gravity-induced falling, we have to find the distributions of temperature, velocity, and pressure in the vicinity of this particle. The general solution of the heat-conduction equation, which satisfies the corresponding boundary conditions, has the form

$$t_e(y,\theta) = t_{e0}(y) = (1 + \Gamma_0/y)^{1/(1+\alpha)}, \tag{2.1}$$

where $t_e = T_e/T_{e\infty}$, $y = r/R$ is the dimensionless radial coordinate, $\Gamma_0 = t_S^{1+\alpha} - 1$ is a dimensionless parameter characterizing the difference in temperature between the particle surface and the medium far from the particle, and $t_S = T_{eS}/T_{e\infty}$. With allowance for Eq. (2.1), we obtain

$$\mu_e = \mu_\infty (1 + \Gamma_0/y)^{\beta/(1+\alpha)}. \tag{2.2}$$

In what follows, relation (2.2) is used to find the velocity and pressure fields in the vicinity of the particle.

The form of the boundary conditions (1.2), (1.3) allows us to separate the variables in solving gas-dynamic equations. The velocity and pressure components are found in the form

$$U_r^e(y,\theta) = U_\infty G(y)\cos\theta, \qquad U_\theta^e(y,\theta) = -U_\infty g(y)\sin\theta, \qquad P_e(y,\theta) = P_{e\infty} + h(y)\cos\theta, \tag{2.3}$$

where $G(r)$, $g(r)$, and $h(r)$ are arbitrary functions depending on the radial coordinate r. Substituting Eqs. (2.2) and (2.3) into the Navier–Stokes equation and separating the variables, we obtain the relation

$$\frac{d^3G}{dy^3} + \frac{1}{y}(4+\gamma_1\ell)\frac{d^2G}{dy^2} - \frac{1}{y^2}(4+\gamma_2\ell-\gamma_3\ell^2)\frac{dG}{dy} - \frac{1}{y^3}(2-\ell)\gamma_3\ell^2 G = -\frac{D}{y^4 t_{e0}^\beta}, \tag{2.4}$$

where

$$\gamma_1 = \frac{1-\beta}{1+\alpha}, \quad \gamma_2 = 2\,\frac{1+\beta}{1+\alpha}, \quad \gamma_3 = \frac{2+2\alpha-\beta}{(1+\alpha)^2}, \quad D = \text{const}, \quad \ell(y) = \frac{\Gamma_0}{y+\Gamma_0}.$$

We seek for the solution of Eq. (2.4) in the form of a generalized power series [13, 14]. First we find the solution of the homogeneous equation (2.4):

$$\frac{d^3G}{dy^3} + \frac{1}{y}(4+\gamma_1\ell)\frac{d^2G}{dy^2} - \frac{1}{y^2}(4+\gamma_2\ell-\gamma_3\ell^2)\frac{dG}{dy} - \frac{1}{y^3}(2-\ell)\gamma_3\ell^2 G = 0. \tag{2.5}$$

For the homogeneous equation (2.5), the point $y = 0$ is a regular singular point [13]. The solution of this equation is sought in the form

$$G = y^\rho \sum_{n=0}^{\infty} C_n \ell^n, \qquad C_0 \neq 0. \tag{2.6}$$

Substituting Eq. (2.6) into Eq. (2.5), we obtain the governing equation $\rho(\rho+3)(\rho-2) = 0$ whose roots are $\rho_1 = -3$, $\rho_3 = 0$, and $\rho_2 = 2$. The following solution corresponds to the greatest root:

$$G_1 = \frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)}\ell^n, \qquad C_0^{(1)} = 1.$$

The second solution of the homogeneous equation (2.5), which satisfies the condition of finiteness for $y \to \infty$, and the particular solution of Eq. (2.4) are found in the form

$$G_3 = \sum_{n=0}^{\infty} C_n^{(3)}\ell^n + \omega_3 \ln y\,\frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)}\ell^n, \qquad G_2 = \frac{1}{y}\sum_{n=0}^{\infty} C_n^{(2)}\ell^n + \frac{\omega_2}{y^3}\,\ln y\sum_{n=0}^{\infty} C_n^{(1)}\ell^n.$$

The expressions for the coefficients $C_n^{(1)}$ $(n \geqslant 1)$, $C_n^{(2)}$ $(n \geqslant 3)$, and $C_n^{(3)}$ $(n \geqslant 4)$ found by the method of undetermined coefficients have the form

$$\begin{aligned}
C_n^{(1)} = \frac{1}{n(n+3)(n+5)}\Big\{&[(n-1)(3n^2+13n+8) + \gamma_1(n+2)(n+3) + \gamma_2(n+2)]C_{n-1}^{(1)} \\
&- [(n-1)(n-2)(3n+5) + 2\gamma_1(n^2-4) + \gamma_2(n-2) + \gamma_3(n+3)]C_{n-2}^{(1)}c \\
&+ (n-2)[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3]C_{n-3}^{(1)}\Big\},
\end{aligned}$$

$$\begin{aligned}
C_n^{(2)} = \frac{1}{(n+1)(n+3)(n-2)}\Big(&[(n-1)(3n^2+n-6) + \gamma_1 n(n+1) + n\gamma_2]C_{n-1}^{(2)} \\
&- [\gamma_3(n+1) + (n-1)(n-2)(3n-1) + 2\gamma_1 n(n-2) + \gamma_2(n-2)]C_{n-2}^{(2)} \\
&+ (n-2)[(n-1)(n-3) + \gamma_3 + \gamma_1(n-3)]C_{n-3}^{(2)} \\
+ \frac{\omega_2}{\Gamma_0^2}\sum_{k=0}^{n-2}(n-k-1)\Big\{&(3k^2+16k+15)C_k^{(1)} - [(k-1)(6k+13) + \gamma_1(2k+5) + \gamma_2]C_{k-1}^{(1)} \\
&+ [3(k-1)(k-2) + 2\gamma_1(k-2) + \gamma_3]C_{k-2}^{(1)}\Big\} - 6(-1)^n\frac{\omega_0!}{n!(\omega_0-n)!}\Big),
\end{aligned}$$

$$C_n^{(3)} = \frac{1}{n(n+2)(n-3)}\Big((n-1)(3n^2-5n-4+\gamma_1 n+\gamma_2)C_{n-1}^{(3)}$$

$$-\,[(n-1)(n-2)(3n-4)+2\gamma_1(n-1)(n-2)+\gamma_2(n-2)+n\gamma_3]C_{n-2}^{(3)}$$

$$+\,(n-2)[(n-1)(n-3)+\gamma_1(n-3)+\gamma_3]C_{n-3}^{(3)}+\frac{\omega_3}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)$$

$$\times\Big\{(3k^2+16k+15)C_k^{(1)}-[(k-1)(6k+13)+\gamma_1(2k+5)+\gamma_2]C_{k-1}^{(1)}+[3(k-1)(k-2)+2\gamma_1(k-2)+\gamma_3]C_{k-2}^{(1)}\Big\}\Big).$$

In calculating the coefficients $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ by recurrent formulas, one has to take into account that

$$C_0^{(1)}=1,\quad C_0^{(3)}=1,\quad C_1^{(3)}=0,\quad C_2^{(3)}=\gamma_3/4,\quad C_3^{(3)}=1,$$

$$\omega_3/(2\Gamma_0^3)=-\gamma_3(10+3\gamma_1+\gamma_2)/60,\quad C_0^{(2)}=1,\quad C_2^{(2)}=1,$$

$$\omega_2/\Gamma_0^2=[(2\gamma_1+\gamma_2+6\omega_0)(4+3\gamma_1+\gamma_2)/4+3\gamma_3+3\omega_0(\omega_0-1)]/15,$$

$$C_1^{(2)}=-(2\gamma_1+\gamma_2+6\omega_0)/8,\quad \omega_0=\beta/(1+\alpha).$$

The function $g(y)$ in the expression for U_θ^e is related to the function $G(y)$ by the functional relation

$$g(y)=G(y)+\frac{1}{2}\,y\Big(\frac{dG(y)}{dy}-fG(y)\Big)\qquad\Big(f=\frac{1}{t_{e0}}\,\frac{dt_{e0}}{dy}=-\frac{\ell}{y(1+\alpha)}\Big),$$

which follows from the continuity equation [second expression in Eqs. (1.1)] with allowance for the temperature dependence of the gaseous medium density ($\rho_e=1/t_{e0}$).

Thus, we obtain the expressions for the velocity components

$$U_r^e=U_\infty\cos\theta\,(A_1G_1+A_2G_2+G_3); \tag{2.7}$$

$$U_\theta^e=-U_\infty\sin\theta\,(A_1G_4+A_2G_5+G_6), \tag{2.8}$$

where

$$G_4=\Big(1+\frac{\ell}{2(1+\alpha)}\Big)G_1+\frac{1}{2}\,yG_1',\qquad G_5=\Big(1+\frac{\ell}{2(1+\alpha)}\Big)G_2+\frac{1}{2}\,yG_2',$$

$$G_6=\Big(1+\frac{\ell}{2(1+\alpha)}\Big)G_3+\frac{1}{2}\,yG_3',$$

G_1', G_2', and G_3' are the first derivatives of the functions G_1, G_2, and G_3 with respect to y.

The constants of integration A_1 and A_2 are determined by substituting expressions (2.7) and (2.8) into the corresponding boundary conditions on the particle surface. The force acting on the particle is found by integrating the stress tensor over the particle surface [12]:

$$\boldsymbol{F}_\mu=\int_S(-P_e\cos\theta+\sigma_{rr}\cos\theta-\sigma_{r\theta}\sin\theta)r^2\sin\theta\,d\theta\,d\varphi\,\boldsymbol{n}_z.$$

Here $\sigma_{rr}=2\mu_e\,\partial U_r^e/\partial r$ and $\sigma_{r\theta}=\mu_e(\partial U_\theta^e/\partial r+(1/r)\,\partial U_r^e/\partial\theta-U_\theta^e/r)$ are the components of the stress tensor in the spherical coordinate system [12], and $\boldsymbol{n}_z$ is the unit vector directed along the Oz axis in the Cartesian coordinate system.

Based on the relations discussed above, we obtain

$$\boldsymbol{F}_\mu=6\pi R\mu_{e\infty}f_\mu U_\infty\boldsymbol{n}_z, \tag{2.9}$$

where $f_\mu=2N_2/(3N_1)$, $N_1\Big|_{y=1}=G_1G_2'-G_2G_1'$, and $N_2\Big|_{y=1}=G_1G_3'-G_3G_1'$.

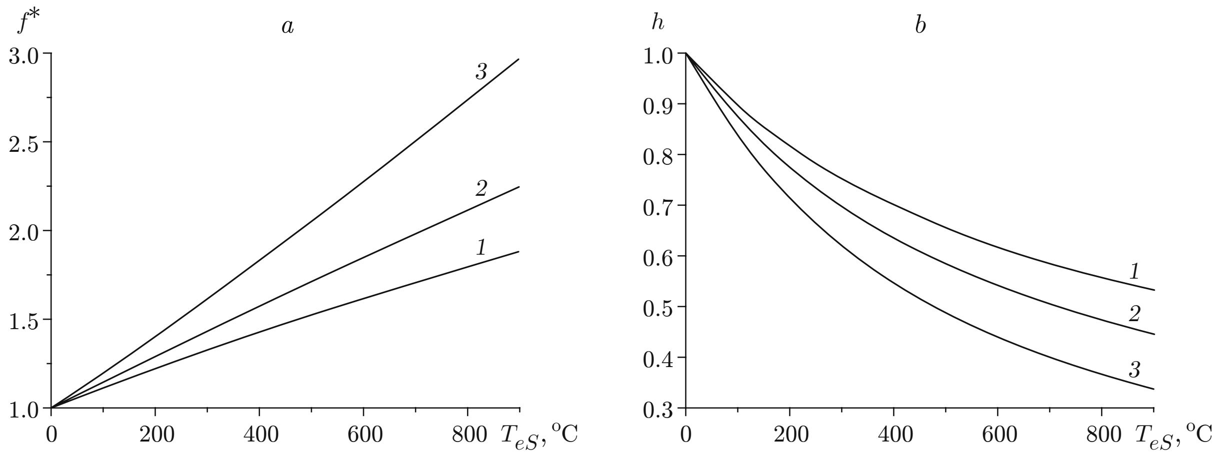

Fig. 1. Dependences of f^* (a) and h (b) on the mean surface temperature T_{eS} for $T_{e\infty} = 0°$C, $P_e = 1$ atm, and $\alpha = \beta = 0.5$ (1), $\alpha = \beta = 0.7$ (2), and $\alpha = \beta = 1.0$ (3).

A spherical particle falling in a viscous medium under the action of the gravity force starts moving with a constant velocity at which the action of the gravity force is balanced by hydrodynamic forces. With allowance for the buoyancy force, the gravity force acting on the particle is

$$\boldsymbol{F} = (\rho_p - \rho_e)g(4/3)\pi R^3 \boldsymbol{n}_z \tag{2.10}$$

(g is the acceleration due to the gravity force; the subscript p refers to the particle).

Equating expressions (2.9) and (2.10), we obtain the expression for the velocity of gravity-induced motion of a uniformly heated spherical particle (an analog of the Stokes formula):

$$U_e = h_\mu \boldsymbol{n}_z, \qquad h_\mu = \frac{2}{9} R^2 \frac{\rho_p - \rho_e}{\mu_{e\infty} f_\mu} g. \tag{2.11}$$

Thus, relations (2.9) and (2.11) allow us to estimate the force acting on a uniformly heated sphere and the velocity of gravity-induced motion of this sphere with allowance for the temperature dependences of the gaseous medium density, viscosity, and thermal conductivity with arbitrary differences in temperature between the particle surface and the ambient medium far from the particle.

If the particle-surface heating is rather low, i.e., the mean temperature of the particle surface differs insignificantly from the ambient temperature far from the particle ($\ell \to 0$), the temperature dependences of density, viscosity, and thermal conductivity may be neglected. Then, we have $G_1 = 1$, $G'_1 = -3$, $G_2 = 1$, $G'_2 = -1$, $G_3 = 1$, $G'_3 = 0$, $N_1 = 2$, and $N_2 = 3$. In this case, relations (2.9) and (2.11) transform to the known Stokes equations for a sphere [11].

Figure 1 shows the dependences of $f^* = f_\mu / f_\mu|_{T_{e\infty}}$ and $h = h_\mu / h_\mu|_{T_{e\infty}}$ on the mean surface temperature T_{eS} for an alumina particle of radius $R = 5$ μm moving in air at $T_{e\infty} = 0°$C and $P_e = 1$ atm. It is seen that particle-surface heating produces a significant effect on the velocity of its gravity-induced motion and on the drag force of the ambient medium. The theoretical conclusions are supported by experimental data (see, e.g., [15]).

Thus, with allowance for the temperature dependences of the density, viscosity, and thermal conductivity of a gaseous medium, expressions are derived, which generalize the Stokes formula to the case of steady motion of a uniformly heated spherical solid particle in a nonisothermal gaseous medium in the field of the gravity force with arbitrary differences in temperature between the particle surface and the ambient medium far from the particle.

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